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MISCELLANEOUS.

73. Proposed by **CHARLES E. MYERS**, Canton, Ohio (February, 1900. p. 54).

In an ice cream freezer, cream of a homogeneous character and at the uniform temperature of 60° F. is put into a cylinder having a closed base, and the whole put into a freezing mixture so as to subject the base and convex surface to a constant temperature of 30° F. Required the temperature at any point within the cream after the expiration of a given time.

Solution by **F. H. SAFFORD**, Ph. D., Instructor in Mathematics in the University of Pennsylvania.

In cylindrical coördinates the equation for the flow of heat is

$$D_t u = c^2 [D_r{}^2 u + \frac{1}{r} D_r u + \frac{1}{r^2} D_\phi{}^2 u + D_z{}^2 u]$$

[Byerly's *Fourier Series*, p. 1]. The axis of z is to be the axis of the cylinder. The planes of the upper and lower bases are respectively $z=0$ and $z=b$, and the radius of the cylinder is a. By the nature of the problem, the temperature u at any time t is independent of ϕ, so that the term above which involves ϕ is to be omitted. From Fourier's *Analytical Theory of Heat*, pp. 51, 113, $c^2 = K/CD$, where D is density of solid (cream), C is specific capacity, and K is specific conductibility.

Let k=temperature of medium surrounding sides and base, $k+m$=internal temperature when $t=0$; also let $v=m$ when $t=0$ .... (1).

The differential equation becomes

$$D_t v = c^2 (D_r{}^2 v + \frac{1}{r} D_r v + D_z{}^2 v) \text{ .... } (2).$$

In order to express the surface conditions due to the surrounding mixture we have two more differential equations

$$D_r u + h(u-k) = 0 \text{ when } r=a,$$
$$D_z u + h(u-k) = 0 \text{ when } z=b.$$

These become

$$D_r v + hv = 0 \text{ when } r=a \text{ .... } (3),$$
$$D_z v + hz = 0 \text{ when } z=b \text{ .... } (4),$$

and are explained in Fourier, p. 94. h is the constant of surface transmission. The value of v must satisfy conditions (1) .... (4). Assume that $v=T.R.Z$ where T, R, and Z are functions of t, r, and z, respectively:

$$\therefore v = m \sum_{\kappa=1}^{\kappa=\infty} \sum_{\lambda=1}^{\lambda=\infty} \varepsilon^{-c^2 t(\mu_\kappa{}^2 + \rho_\lambda{}^2)} . A_\kappa J_0 (\mu_\kappa r) . B_\lambda \sin \rho_\lambda z \text{ .... } (5).$$

(For method, see Byerly, p. 195). J_0 is a Bessel Function while μ_κ and ρ_λ are to be determined by (3) and (4) which give, after v is substituted from (5),

$$\mu_\kappa aJ_1(\mu_\kappa a)-ahJ_0(\mu_\kappa a)=0 \text{ ....} (6),$$
$$\rho_\lambda b\cos\rho_\lambda b+bh\sin\rho_\lambda b=0 \text{ ....} (7).$$

It is shown in Riemann (*Partial Differential Equations*, §97) that (6) has an infinite number of real positive roots, while tables and a diagram in Gray and Mathew's *Bessel's Functions* make their calculation easy. Fourier, p. 270, states the same property for (7).

We use condition (1) in determing A_κ and B_λ, as when $t=0$ (5) must give

$$m=m\sum_{\kappa=1}^{\kappa=\infty}\sum_{\lambda=1}^{\lambda=\infty} A_\kappa J_0(\mu_\kappa r)B_\lambda \sin\rho_\lambda z.$$

It remains to obtain values of A_κ and B_λ such that

$$\sum_{\kappa=1}^{\kappa=\infty} A_\kappa J_0(\mu_\kappa r)=1, \qquad \sum_{\lambda=1}^{\lambda=\infty} B_\lambda \sin\rho_\lambda z=1.$$

From Byerly, pp. 229, 121, we find

$$A_\kappa=\frac{2h}{a(h^2+\mu_\kappa^2)J_0(\mu_\kappa a)} \text{ ....} (8),$$

$$B_\lambda=\frac{2b}{\rho_\lambda}\cdot\frac{\rho_\lambda^2+h^2}{\rho_\lambda^2b^2+bh(bh+1)}(1-\cos\rho_\lambda b) \text{ ....} (9).$$

The same excellent authority gives the means of solving cases in which the initial temperature is much more complicated than in the present problem.

In conclusion, the solution of the given problem is

$$u=k+m\sum_{\kappa=1}^{\kappa=\infty}\sum_{\lambda=1}^{\lambda=\infty} \varepsilon^{-c^2t(\mu_\kappa^2+\rho_\lambda^2)}.A_\kappa J_0(\mu_\kappa r).B_\lambda \sin\rho_\lambda z,$$

the constants being determined by (6), (7), (8), (9).